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Degenerate and inverted hierarchical models of Majorana neutrinos from see-saw mechanism

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Abstract In this communication, we generate the textures of the nearly degenerate as well as the inverted hierarchical models of the left-handed Majorana neutrino mass matrices within the framework of the see-saw mechanism in a model independent way. The leptonic mixings are generated from the texture of the right-handed Majorana mass matrices while keeping the Dirac neutrino mass matrices in diagonal forms. Such textures of the Majorana neutrino mass matrices are important to explain the recently reported result on the double beta decay ($0\nu\beta\beta$) experiment, together with the earlier established data on LMA MSW solar and atmospheric neutrino oscillations.

Keywords Majorana neutrino mass, degenerate and inverted hierarchical models, see-saw mechanism

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1. Introduction

The recently reported experimental result on double beta decay ($0\nu\beta\beta$) shows the possible evidence for the non-zero Majorana mass of the electron neutrino in the range of $m_{ee} = (0.05 - 0.86)$ eV at 95% C.L. with the best value of $m_{ee} = 1.4$ eV[1]. There are certain important implications of this result. Firstly, it rules out all models predicting the Dirac neutrino masses, leaving only the option for the Majorana type of neutrinos. It also allows the lepton number violating processes such as leptogenesis in a natural way. Secondly, this result together with the earlier experimental data on the atmospheric [2] and the solar [3] neutrino oscillations, allows only the degenerate and the inverted hierarchical solutions for the three generation left-handed Majorana neutrinos [1,4].

In the above context, it is important to construct theoretical models which can predict the degenerate and inverted hierarchical patterns of the Majorana neutrino mass matrices within the framework of the grand unified theories (GUTs) with or without supersymmetry [4, 5]. In this communication, we attempt to generate the degenerate as well as the inverted hierarchical pattern of the left-handed Majorana neutrino mass matrices using the see-saw formula in a model independent way. This is, in fact, a continuation of our earlier work [6] where the neutrino mixings are provided from the texture of the right-handed Majorana mass matrix M_{RR}, while keeping the Dirac neutrino mass matrix m_{LR} in the diagonal form. We had taken the Dirac neutrino mass matrix m_{LR} as either the charged lepton mass matrix ($m_{LR} = \tan\beta \; m_l$ referred to as case (i)) or the up-quark mas matrix ($m_{LR} = m_{up}$ referred to as case (ii)) [7]. While referring to the earlier paper [6] for details, we point out that the model successfully generated both the hierarchical and the inverted hierarchical (having opposite sign mass eigenvalues) neutrino mass matrices as a result of the proper choice of the parameters in texture of M_{RR}. In Section 2 we present the generation of the degenerate as well as inverted hierarchical neutrino mass matrices using the seesaw formula, and their predictions on mass eigenvalues and mixing angles. Section 3 is devoted to summary and conclusion.

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2. Neutrino mass matrices from see-saw formula

The left-handed Majorana neutrino mass matrix m_{LL} is given by the celebrated see-saw formula [8],

$$m_{LL} = -m_{LR} M_{RR}^{-1} m_{LR}^{T}, \tag{1}$$

where m_{LR} is the Dirac neutrino mass matrix in the left-right (LR) convention [9] The leptonic (MNS) mixing matrix is now given by $V_{MNS} = V_{\nu L}^{\dagger}$ where $m_{LL}^{diag} = V_{\nu L}\, m_{LL}\, V_{\nu l}^{T}$. Here, both m_{LR} and the charged lepton mass matrix m_l are taken as diagonal, whereas the right-handed Majorana neutrino mass matrix M_{RR} as non-diagonal Using the see-saw formula (1) we generate both patterns of m_{LL} viz., (I) nearly degenerate and (II) inverted hierarchical neutrino mass matrices with same sign mass eigenvalues. We concentrate here only on the cases which have bimaximal mixings listed in Table 1.

Table 1. Zeroth order neutrino mass matrices with texture zeros corresponding to the LMA MSW solution with bimaximal mixings [4, 10]

Type	m_{LL}	m_{LL}^{diag}
I(A)	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag\,(1,\ 1,\ 1)\ m_0$
I(B)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0$	$Diag\,(1,\ 1\ \ 1)\ m_0$
I(C)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$Diag\,(1,\ 1,\ -1)\ m_0$
II(A)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$Diag\,(1,\ 1,\ 0)\ m_0$
II(B)	$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0$	$Diag\,(1,\ -1,\ 0)\ m_0$

I(A) Nearly degenerate mass matrix with opposite sign mass eigenvalues.

We consider the following form of m_{LL} with opposite mass eigenvalues

$$m_{LL} = \begin{pmatrix} -2\delta_1 + 2\delta_2 & \frac{1}{\sqrt{2}} - \delta_1 & \frac{1}{\sqrt{2}} - \delta_1 \\ \frac{1}{\sqrt{2}} - \delta_1 & \frac{1}{2} + \delta_2 & \frac{1}{2} + \delta_2 \\ \frac{1}{\sqrt{2}} - \delta_1 & -\frac{1}{2} + \delta_2 & \frac{1}{2} + \delta_2 \end{pmatrix} m_0 \qquad (2)$$

where m_0 controls the overall magnitude of the masses of the neutrinos whereas δ_1 and δ_2 give the desired splittings for solar and atmospheric data. When , $\delta_1 = \delta_2 = 0$ eq. (2) reduces to the zeroth order mass matrix of the type I(A) in Table-1, with no splittings [10].

The diagonalisation of m_{LL} in eq. (2) leads to [see Appendix]

$$m_{\nu 1} = \left(1 + 2\delta_2 - \delta_1(1+\sqrt{2})\right) m_0,$$

$$m_{\nu 2} = \left(-1 + 2\delta_2 - \delta_1(1-\sqrt{2})\right) m_0,$$

$$\sin^2 2\theta_{12} \approx \left(1 - \delta_1^2/8\right), \sin^2 2\theta_{23} = 1,\ \sin^2 2\theta_{13} = 0$$

For the choice [5] of the values of the parameters $m_0 = 0.4$ eV $\delta_1 = 0.0061875$, $\delta_2 = 0.0030625$, eq. (2) leads to the following numerical predictions :

Mixing angles :

$$\sin^2 2\theta_{12} = 0.999,\ \sin^2 2\theta_{23} \approx 1.0,\ |V_{e3}| = 6.124 \times 10^{-3}$$

Mass eigenvalues :

$$m_{\nu i} = (0.396484, -0.396532, 0.4)\ eV,\ i = 1, 2, 3,$$

$$\Delta m_{12}^2 = 3.806 \times 10^{-5} eV^2 \text{ and } \Delta m_{23}^2 = 2.76 \times 10^{-3} eV^2$$

The prediction on solar mixing angle is consistent with the LMA MSW solution [3].

Case (i) where $m_{LR} = \tan\beta\, m_l$ [6,7]

The degenerate mass matrix m_{LL} in eq.(2) is now generated through the see-saw formula [8] in eq.(1) for the following choices of m_{LR} and M_{RR} respectively :

$$m_{LR} = \tan\beta \begin{matrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^6 & 0 \\ 0 & 0 & 1 \end{matrix}\ m_\tau$$

and

$$M_{RR} = \begin{matrix} -2\delta_2\lambda^{12} & \left(\frac{1}{\sqrt{2}} + \delta_1\right)\lambda^8 & \left(\frac{1}{\sqrt{2}} + \delta_1\right)\lambda^6 \\ \left(\frac{1}{\sqrt{2}} + \delta_1\right)\lambda^8 & \left(\frac{1}{2} + \delta_1 - \delta_2\right)\lambda^4 & \left(-\frac{1}{2} + \delta_1 - \delta_2\right)\lambda^2 \\ \left(\frac{1}{\sqrt{2}} + \delta_1\right)\lambda^6 & \left(-\frac{1}{2} + \delta_1 - \delta_2\right)\lambda^2 & \frac{1}{2} + \delta_1 - \delta_2 \end{matrix}$$

The inverse of M_{RR} has a simple form for $\delta_2 > \delta_1$ and $\delta_1, \delta_2 << 1$,

$M_{RR}^{-1} =$

$$\begin{pmatrix} (1-2\delta_1+2\delta_2)\lambda^{-12} & \left(\frac{1}{\sqrt{2}}-\delta_1\right)\lambda^{-8} & \left(\frac{1}{\sqrt{2}}-\delta_1\right)\lambda^{6} \\ \left(\frac{1}{2}-\delta_1\right)\lambda^{-8} & \left(\frac{1}{2}+\delta_2\right)\lambda^{-4} & \left(-\frac{1}{2}+\delta_2\right)\lambda^{-2} \\ \left(1-\delta_1\right)\lambda^{6} & \left(-\frac{1}{2}+\delta_2\right)\lambda^{-2} & \left(\frac{1}{2}+\delta_2\right) \end{pmatrix} \left(v_R^{-1}\right).$$

The expression for m_0 in eq. (2) is worked out as $m_0 = m_\tau^2 \tan^2\beta / v_R$. For input values of $m_0 = 0.4$ eV, $\tan\beta = 40$, $m_\tau = 1.7$ GeV, and $\lambda = 0.22$, the see-saw scale is calculated as $v_R = 10^{13}$ GeV. This in turn, gives the masses of the three right-handed Majorana neutrinos after the diagonalisation of M_{RR}. $|M_{RR}^{diag}| = (5.0427\times10^{12}, 3.0981\times10^{8}, 1.9613\times10^{7})$ GeV.

Case (ii) where $m_{LR} = m_{up}$ [6, 7]

The texture of m_{LL} in eq. (2) is again realised through the see-saw formula (1) with the following textures of m_{LR} and

$$m_{LR} = \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_t, \tag{5}$$

$M_{RR} =$

$$\begin{pmatrix} -2\delta_1\lambda^{16} & \left(\frac{1}{\sqrt{2}}+\delta_1\right)\lambda^{12} & \left(\frac{1}{\sqrt{2}}+\delta_1\right)\lambda^{8} \\ \left(\frac{1}{\sqrt{2}}+\delta_1\right)\lambda^{12} & \left(\frac{1}{2}+\delta_1-\delta_2\right)\lambda^{8} & \left(-\frac{1}{2}+\delta_1-\delta_2\right)\lambda^{4} \\ \left(\frac{1}{\sqrt{2}}+\delta_1\right) & \left(-\frac{1}{2}+\delta_1-\delta_2\right)\lambda^{4} & \left(\frac{1}{2}+\delta_1-\delta_2\right) \end{pmatrix} v_R. \tag{6}$$

We have $m_0 = m_t^2 / v_R$ in eq. (2), and with the input values $m_0 = 0.4$ eV $m_t = 200$ GeV, we obtain $v_R = 10^{14}$ GeV and the mass eigenvalues of the right-handed Majorana neutrinos : $|M_{RR}^{diag}|$ $= (4.5932\times10^{15}, 7.2731\times10^{6}, 5.005\times10^{13})$ Gev.

I(B) Nearly degenerate mass matrix with same sign mass eigenvalues .

We propose another form of the nearly degenerate mass matrix.

$$\begin{pmatrix} (1-2\delta_1-2\delta_2) & -\delta_1 & -\delta_1 \\ -\delta_1 & (1-\delta_2) & -\delta_2 \\ -\delta_1 & -\delta_2 & (1-\delta_2) \end{pmatrix} m_0. \tag{7}$$

The diagonalisation of m_{LL} in eq. (7) leads to [see Appendix]

$$m_{\nu 1} = \left(1-2\delta_2-(\sqrt{3}+1)\delta_1\right)m_0,$$

$$m_{\nu 2} = \left(1-2\delta_2+(\sqrt{3}-1)\delta_1\right)m_0,$$

$$m_{\nu 3} = m_0,$$

$$\sin^2 2\theta_{12} = \frac{2}{3},\ \sin^2 2\theta_{23} = 1,\ \sin^2 2\theta_{13} = 0.$$

For the choice [5] of the values of the parameters $m_0 = 0.4$ eV, $\delta_1 = 3.6\times10^{-5}$, $\delta_2 = 3.9\times10^{-3}$, eq. (2) leads to the following numerical predictions :

Mixing angles :

$$\sin^2 2\theta_{12} = 0.67,\ \sin^2 2\theta_{23} \approx 1.0, |V_{e3}| = 1.5\times10^{14},$$

Mass eigenvalues :

$$m_{i} = (0.39684, 0.396892, -0.4)\,\text{eV}, i = 1, 2, 3;$$

$$\Delta m_{12}^2 = 4.13\times10^{-5}\ \text{eV}^2 \text{ and } \Delta m_{23}^2 = 2.48\times10^{-3}\ \text{eV}^2.$$

The prediction on solar mixing angle is consistent with the LMA MSW solution [3]

This form of the mass matrix (7) can be realised in the see-saw mechanism (1) using the following textures of m_{LR} and M_{RR}.

Case (i) where $m_{LR} = \tan\beta\, m_l$

Here, m_{LR} is given in eq. (3) and the right-handed neutrino mass matrix takes the form :

$$M_{RR} = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{12} & \delta_1\lambda^8 & \delta_1\lambda^6 \\ \delta_1\lambda^8 & (1+\delta_2)\lambda^4 & \delta_2\lambda^2 \\ \delta_1\lambda^6 & \delta_2\lambda^2 & (1+\delta_2) \end{pmatrix} v_R. \tag{8}$$

The inverse of M_{RR} has a simple form for $\delta_2 > \delta_1$ and $\delta_1, \delta_2 << 1$.

$$M_{RR}^{-1} = \begin{pmatrix} (1-2\delta_1-2\delta_2)\lambda^{-12} & -\delta_1\lambda^{-8} & -\delta_1\lambda^{-6} \\ -\delta_1\lambda^{-8} & (1-\delta_2)\lambda^{-4} & -\delta_2\lambda^{-2} \\ -\delta_1\lambda^{-6} & -\delta_2\lambda^{-2} & (1-\delta_2) \end{pmatrix} \left(v_R^{-1}\right)$$

The expression for m_0 in eq. (7) is again worked out as $m_0 = m_\tau^2 \tan^2\beta / v_R$. For input values of $m_0 = 0.4$ eV, $\tan\beta = 40$, $m_\tau = 1.7$ GeV, we obtain $v_R = 1.156\times10^{13}$ GeV which leads to $M_R^{diag} = (1.15\times10^{13}, 2.71\times10^{10}, 1.498\times10^{5})$ GeV.

Case (ii) where $m_{LR} = m_{up}$

The texture of m_{LL} is given in eq. (5), and the texture of M_{RR} has the form :

$$M_{RR} \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{16} & \delta_1\lambda^{12} & \delta_1\lambda^8 \\ \delta_1\lambda^{12} & (1+\delta_2)\lambda^8 & \delta_2\lambda^4 \\ \delta_1\lambda^8 & \delta_2\lambda^4 & (1+\delta_2) \end{pmatrix} \tag{9}$$

and $m_0 = m_t^2 / v_R$.

For input values of $m_0 = 0.4$ eV and $m_t = 200$ GeV, we have $v_R = 1.0\times 10^{14}$ GeV and $M_{RR}^{diag} = (1.0039\times 10^{14}, 5.5089\times 10^8, 3.035\times 10^4)$ GeV.

I(C). Nearly degenerate mass matrix with opposite sign mass eigenvalues .

We consider another texture for the nearly degenerate mass matrix m_{LL} with opposite mass eigenvalues represented by [5]

$$m_{LL} = \begin{pmatrix} (1-2\delta_1-2\delta_2) & -\delta_1 & -\delta_1 \\ -\delta_1 & -\delta_2 & (1-\delta_2) \\ -\delta_1 & (1-\delta_2) & -\delta_2 \end{pmatrix} m_0 \tag{10}$$

where m_0 controls the overall magnitude of the masses of the neutrinos and δ_1 and δ_2 give the desired splittings for solar and atmospheric data. When $\delta_1 = \delta_2 = 0$ eq. (10) reduces to the zeroth order mass matrix of the Type I(C) in Table-1, with no splittings [5, 10].

The diagonalisation of m_{LL} in eq. (10) leads to the following eigenvalues and mixing angles [see Appendix] :

$$m_{\nu 1} = \left(1-2\delta_2-(\sqrt{3}+1)\delta_1\right)m_0,$$

$$m_{\nu 2} = \left(1-2\delta_2+(\sqrt{3}-1)\delta_1\right)m_0,$$

$$m_{\nu 3} = -m_0$$

$$\sin^2 2\theta_{12} = \frac{2}{3},\ \sin^2 2\theta_{23} = 1,\ \sin^2 2\theta_{13} = 0 .$$

The numerical solution leads to m_{ν_i} =(0.39684, 0.396892, 0.4) eV, $i = 1, 2, 3$ for the same choices of the input values of $\delta_{1,2}$ and m_0 as in eq. (7). Further, the predictions on the three mixing angles are the same as in eq. (7). When $\delta_1 = \delta_2 = 0$, it reduces to the type I(B) in the Table 1.

Case (i) where $m_{LR} = \tan\beta\, m_l$ [6, 7]

We consider m_{LR} given in eq. (3) and then the right-handed neutrino mass matrix takes the form

$$M_{RR} = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{12} & \delta_1\lambda^8 & \delta_1\lambda^6 \\ \delta_1\lambda^8 & \delta_2\lambda^4 & (1+\delta_2)\lambda^2 \\ \delta_1\lambda^6 & (1+\delta_2)\lambda^2 & \end{pmatrix} v_R \tag{11}$$

The inverse of M_{RR} has a simple form for $\delta_2 > \delta_1$ and $\delta_1, \delta_2 << 1$,

$$M_{RR}^{-1} = \begin{pmatrix} (1-2\delta_1-2\delta_2)\lambda^{-12} & -\delta_1\lambda^{-8} & -\delta_1\lambda^{-6} \\ -\delta_1\lambda^{-8} & -\delta_2\lambda^{-4} & (1-\delta_2)\lambda^{-2} \\ -\delta_1\lambda^{-6} & (1-\delta_2)\lambda^{-2} & -\delta_2 \end{pmatrix}$$

For input values of $m_0 = 0.4$eV, $\tan\beta = 40$, $m_\tau = 1.7$ GeV, and $\lambda = 0.22$, the see-saw scale is calculated as $v_R \approx 10^{13}$ GeV This in turn gives the masses of the three right-handed Majorana neutrinos after the diagonalisation of M_{RR} :

$$M_{RR}^{diag} = (4.67\times 10^{11}, 1.296\times 10^5, 5.06\times 10^4)\,\text{GeV}.$$

Case (ii) where $m_{LR} = m_{up}$ [6, 7]

The texture of m_{LL} in eq. (10) is again realised through the see-saw formula (1) with the textures of m_{LR} as given in eq (5) and M_{RR}

$$M_{RR} = \begin{pmatrix} (1+2\delta_1+2\delta_2)\lambda^{16} & \delta_1\lambda^{12} & \delta_1\lambda^8 \\ \delta_1\lambda^{12} & \delta_2\lambda^8 & (1+\delta_2)\lambda^4 \\ \delta_1\lambda^8 & (1+\delta_2)\lambda^4 & \delta_2 \end{pmatrix} v_R \tag{12}$$

We have $m_0 = m_t^2 / v_R$ in eq. (10), and with the input values $m_0 = 0.4$ eV, $m_t = 200$ GeV, we obtain $v_R = 10^{14}$ GeV and the mass eigenvalues of the right-handed Majorana neutrinos : M_{RR}^{diag} = $(1.105\times 10^{11}, 3.035\times 10^3, 5.005\times 10^{11})$ GeV.

II(A). Inverted hierarchical mass matrix with same sign mass eigenvalues .

The most general form of the inverted hierarchical mass matrix m_{LL} with the same sign mass eigenvalues can be expressed as

$$m_{LL} = \begin{pmatrix} (1-2\varepsilon) & -\varepsilon & -\varepsilon \\ -\varepsilon & a & (a-\eta) \\ -\varepsilon & (a-\eta) & a \end{pmatrix} m_0', \tag{13}$$

where $a = 0.5$ and m_0' is the overall factor for the masses of the neutrinos. The parameters ε and η give the desired splittings for solar and atmospheric data.

The diagonalisation of m_{LL} in eq. (13) leads to the following eigenvalues and mixing angles [see Appendix] :

$$m_{\nu_1} = \left(1-\left(\sqrt{3}+1\right)\varepsilon-\frac{\eta}{2}+\frac{\sqrt{\eta}\varepsilon}{6}\right)m_0',$$

$$m_{\nu_2} = \left(1+\left(\sqrt{3}-1\right)\varepsilon-\frac{\eta}{2}-\frac{\sqrt{\eta\varepsilon}}{6}\right)m_0',$$

$$m_{\nu_3} = \eta m_0',$$

and mixing angles :

$$\sin^2 2\theta_{12} = \frac{2}{3},\ \sin^2 2\theta_{23} = 1,\ \sin^2 2\theta_{13} = 0.$$

When $\varepsilon = \eta = 0$ eq. (13) reduces to the zeroth order mass matrix of the type II(A) in Table-I, with no solar splitting [4, 10]. For solution of the LMA MSW solar data and atmospheric neutrino oscillation, we have the choice of the parameters $m_0' = 0.05$ eV, $\varepsilon = 0.002$ and $\eta = 0.0001$ leading to the following predictions .

Mixing angles :

$\sin^2 2\theta_{12} = 0.67$, $\sin^2 2\theta_{23} \approx 1.0$, $|V_{e3}| = 3.04\times 10^{-13}$,

Mass eigenvalues :

$m_i = (0.05007, 0.04973, 0.000005)$ eV, $i = 1, 2, 3$; leading to $\Delta m_{12}^2 = 3.393\times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m_{23}^2 = 2.47\times 10^{-3}\,\mathrm{eV}^2$.

Here the neutrino mass eigenvalues are of the same sign and this differs from the other inverted hierarchical mass matrix having the following form [11],

$$\text{where } m_{LL} = \begin{pmatrix} & 1 & 1 \\ & \delta_1 & \delta_2 \\ & \delta_2 & \delta_1 \end{pmatrix} m_0',\ \varepsilon,\ \delta_1,\delta_2 << 1 \qquad (14)$$

which gives opposite sign mass eigenvalues. For $\delta_1, \delta_2, \varepsilon = 0$, it leads to the type II(B) in Table I. This structure has been successfully generated within this model [6], and the rediative correction has been found to be weak [12, 13]. We shall not address again this model here. Instead, we concentrate on the generation of the texture of m_{LL} given in eq. (10) from the see saw mechanism (1).

Case (i) when $m_{LR} = \tan\beta\, m_l$

The inverted hierarchical mass matrix m_{LL} in eq. (13) can now be realised with the choice of $m_{LR} = \tan\beta\, m_l$ given in eq. (3) and M_{RR} of the following form

$$M_{RR} = \begin{pmatrix} 2a\eta(1+2\varepsilon)\lambda^{12} & \eta\varepsilon\lambda^{8} & \eta\varepsilon\lambda^{6} \\ \eta\varepsilon\lambda^{8} & a\lambda^{4} & -(a-\eta)\lambda^{2} \\ \eta\varepsilon\lambda^{6} & -(a-\eta)\lambda^{2} & a \end{pmatrix} \frac{v_R}{2a\eta}, \qquad (15)$$

where M_{RR}^{-1} has a simple form for $\varepsilon > \eta$ and $\varepsilon, \eta << 1$,

$$M_{RR}^{-1} = \begin{pmatrix} (1-2\varepsilon)\lambda^{-12} & -\varepsilon\lambda^{-8} & -\varepsilon\lambda^{-6} \\ -\varepsilon\lambda^{-8} & a\lambda^{-4} & (a-\eta)\lambda^{-2} \\ -\varepsilon\lambda^{-6} & (a-\eta)\lambda^{-2} & \end{pmatrix} \left(v_R^{-1}\right)$$

The expression for m_0' in eq. (13) is given by $m_0' = m_\tau^2 \tan^2\beta / v_R$. For input values of $m_0' = 0.05$ eV, $\tan\beta = 5$, $m_\tau = 1.7$ GeV, we obtain $v_R = 1.445\times 10^{12}$ GeV which leads to $M_{RR}^{diag} = (9.742\times 10^{3}, 2.831\times 10^{4}, 7.24\times 10^{15})$ GeV.

Case (ii) when $m_{LR} = m_{up}$

The texture of m_{LL} in eq. (13) is again realised through $m_{LR} = m_{up}$ given in eq. (5) and the texture of M_{RR} :

$$M_{RR} = \begin{pmatrix} 2a\eta(1+2\varepsilon)\lambda^{16} & \eta\varepsilon\lambda^{12} & \eta\varepsilon\lambda^{8} \\ \eta\varepsilon\lambda^{12} & a\lambda^{8} & -(a-\eta)\lambda^{4} \\ \eta\varepsilon\lambda^{8} & -(a-\eta)\lambda^{4} & a \end{pmatrix} \frac{v_R}{2a\eta}, \qquad (16)$$

which leads to $m_0' = m_t^2 / v_R$ in eq. (13). Using the input values $m_0' = 0.05$ eV, $m_t = 200$ GeV, we have $v_R = 8\times 10^{14}$ GeV and $\left|M_{RR}^{diag}\right| = (2.4\times 10^4, 4\times 10^{18}, 2.4\times 10^9)$ GeV where the mass of the heaviest right-handed Majorana neutrino lies above the GUT scale but below the Planck scale [10].

A few comments on the stability condition under radiative corrections are in order. The nearly degenerate mass matrices m_{LL} in eqs (2), (7) and (10) are found to be unstable under radiative correction in minimal super-symmetric standard model. The inverted hierarchical mass matrix with the same mass eigenvalues given in eq. (13) is also found to be unstable under radiative correction. However, the inverted hierarchical mass matrix given in eq (14) with opposite sign mass eigenvalues is stable under radiative correction [12, 13]. The radiative stability of the neutrino mass textures remains a very important problem at the moment [14].

3. Summary

In summary, we generate the textures of the nearly degenerate as well as the inverted hierarchical left-handed Majorana neutrino mass matrices from the see-saw formula using the diagonal form of the Dirac mass matrix and non-diagonal form of the right-handed Majorana neutrino mass matrix. The predictions on lepton mixing angles $\sin^2 2\theta_{12} \approx 0.67$, $\sin^2 2\theta_{23} \approx 1.0$ and $|V_{e3}| \approx 0$ are in excellent agreement with the experimental values. This is also true for the predictions of Δm_{12}^2 and Δm_{23}^2 which are necessary for the $0\nu\beta\beta$ decays, LMA MSW solar oscillation and atmospheric oscillation data. In all cases the masses of the right-handed Majorana neutrinos are above the weak scale. An interesting observation is that the prediction on the solar mixing angle is large but not maximal without any extra fine-tuning.

Though the present analysis is a model independent one, it may be a useful guide for building models under the framework of grand unified theories with the extended flavour U(1) symmetry.

References

[1] H V Klapdor-Kleingrothaus, A Dietz, H L Harney and I V Krivosheina, *Mod Phys Lett* **A16** 2409 (2001)

[2] Y Fukuda *et al*, Super-Kamiokande Collaboration, *Phys Rev. Lett.* **85** 3999 (2000)

[3] Y Fukuda *et al*, Super-Kamiokande Collaboration, *Phys Rev Lett.* **86** 5656 (2001) and references therein See also Q R Ahmed *et al*, SNO Collaboration, *Phys Rev Lett* **87** 071301 (2001), A Bandopadhyay, S Choubey, S Goswami and D P Roy *hep-ph/0204286*

[4] H V Klapdor-Kleingrothaus and U Sarkar, **hep-ph/0201224**, A S Joshipura and S D Rindani *hep-ph/0202064*

[5] E Ma *hep-ph/0201225*

[6] N Nimai Singh and Mahadev Patgiri **hep-ph/0111319** (To appear in IJMPA, 2002)

[7] K S Babu, B Dutta and R N Mohapatra *Phys Lett.* **B458** 93 (1999) *hep-ph/9904366*

[8] M Gell-Mann, P Ramond and R Slansky in *Supergravity*, (Amsterdam North-Holland) (1979) ; T Yanagida in *Proc of the Workshop on Unified Theory and Baryon Number of the Universe*, (*KEK, Japan*) (1979), R N Mohapatra and G Senjanovic *Phys. Rev. Lett* **44** 912 (1980)

[9] S F King and N Nimai Singh *Nucl Phys* **B591** 3 (2000), *Nucl. Phys* **B596** 81 (2001)

[10] G Altarelli and F Feruglio *Phys Rep.* **320** 295 (1999)

[11] An incomplete list ; R Barbieri, L Hall, D Smith, A Strumia and N Weiner JHEP **9812** 17 (1998), *hep-hp/9807235*, A S Joshipura and S D Rindani *Eur Phys. J.* **C14** 85 (2000) ; Q Shafi and Z Tavartkiladze *Phys Lett* **B482** 145 (2000) ; S F King and N Nimai Singh *Nucl Phys* **B596** 81 (2001) ; M Patgiri and N Nimai Singh *hep-ph/0112123* ; K S Babu, R N Mohapatra, **hep-ph/0201176**

[12] P H Chankowski, W Krolikowski and S Pokorski *Phys Lett* **B473** 109 (2000), N Haba and N Okamura *Eur Phys J* **C14** 347 (2000)

[13] S F King and N Nimai Singh *Nucl Phys* **B596** 81 (2001), M Patgiri and N Nimai Singh *hep-ph/0112123*

[14] M K Parida, C R Das and G Rajsekaran *hep-hp/0203097*, H V Klapdor-Kleingrothaus and U Sarkar *Mod Phys Lett.* **A16** 2469 (2001)

[15] T Kitabayashi and M Yasue *hep-ph/0112287*

Appendix

Diagonalisation of the mass matrix m_{LL}:

Although the diagonalisation of m_{LL} is trivial [5], we find it convenient to follow the general result given in Ref. [15]. The neutrino mass matrix m_{LL} of the general form

$$m_{LL} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

where $c = -t_{23}b$ and $t_{23} = \sin\theta_{23}/\cos\theta_{23}$,

$$f = d + \left(t_{23}^{-1} - t_{23}\right)e$$

can be diagonalised by V_{MNS}

$$V_{MNS} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & \\ -\cos\theta_{23}\sin\theta_{12} & \cos\theta_{23}\sin\theta_{12} & \sin\theta_{23} \\ \sin\theta_{23}\sin\theta_{12} & -\sin\theta_{23}\sin\theta_{12} & \cos\theta_{23} \end{pmatrix}$$

where we have taken $\sin\theta_{13} = 0$. This mixing MNS matrix transforms $|\nu_i\rangle$ with the mases $(m_{\nu1}, m_{\nu2}, m_{\nu3})$ into

$|\nu_f\rangle = V_{MNS}|\nu_i\rangle$, $f = e, \mu, \tau$ and $i = 1, 2, 3$. The mass eigenvalues and θ_{12} are calculated as

$$m_{\nu1} = a - \frac{1}{2}\sqrt{\frac{b^2+c^2}{2}}\left(x + \eta\sqrt{x^2+8}\right),\ m_{\nu2} = (\eta \to -\eta \text{ in } m_{\nu1}),$$

$$m_{\nu3} = d + t_{23}^{-2}\left(d - a + x\sqrt{\frac{b^2+c^2}{2}}\right),\ \sin^2 2\theta_{12} = \frac{8}{8+x^2},$$

with $x(a - d + t_{23}e)/\left(\sqrt{\frac{b^2+c^2}{2}}\right)$ where $|m_{\nu1}| < |m_{\nu2}|$ is always maintained by adjusting the sign of $\eta(=\pm 1)$.